Solutions to Mock JEE MAIN – 2 |JEE - 2024

Physics

SINGLE CHOICE

1.(B) Since, potential V is same for all points of the sphere. Therefore, we can calculate its value at the centre of the sphere.

$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r} + V'$

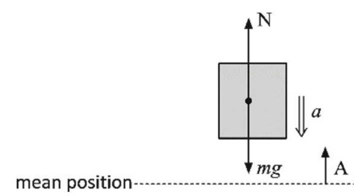
V' = potential at centre due to induced charges = 0 (because net induced charge will be zero)

$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$

2.(A) $A_R = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = 2 \qquad \phi = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3};$

So equation of SHM is : $\quad y = 2\sin\left(\omega t + \frac{\pi}{3}\right)$

Maximum chance of break off is at extreme position. $mg - N = m\omega^2 A$



For breakoff $N = 0 \quad \Rightarrow \quad \omega = \sqrt{\frac{g}{A}} = \sqrt{\frac{g}{2}}$

Also for $y = A = 2 \quad \Rightarrow \quad \omega t + \frac{\pi}{3} = \frac{\pi}{2} \quad \Rightarrow \quad t = \frac{\pi}{6\omega} = \frac{\pi}{6}\sqrt{\frac{2}{g}}$

3.(B) $V = IR$

$$\Rightarrow \quad R = \frac{V}{I} = \frac{U}{qI} = \frac{U}{I^2 t} = \frac{\left(ML^2T^{-2}\right)}{\left[A^2\right]\left[T\right]} = \left[ML^2T^{-3}A^{-2}\right]$$

$$\Rightarrow \quad C = \frac{Q}{V} = \frac{It}{U/q} = \frac{I^2t^2}{U} = \frac{\left[A^2\right]\left[T^2\right]}{\left[ML^2T^{-2}\right]} = \left[M^{-1}L^{-2}T^4A^2\right]$$

$$\Rightarrow \quad F = iBl$$

$$B = \frac{F}{il} = \frac{\left[MLT^{-2}\right]}{\left[A\right]\left[L\right]} = \left[MT^{-2}A^{-1}\right]$$

$$\Rightarrow \quad U = \frac{1}{2}Li^2$$

$$L = \frac{U}{i^2} = \frac{\left[ML^2T^{-2}\right]}{\left[A^2\right]} = \left[ML^2T^{-2}A^{-2}\right]$$

4.(B) $I = \frac{100\times10^{-3}}{0.5}A = 0.2A$

$R = \frac{1}{0.2}\Omega = 5\Omega.$

5.(B) Comparing the given equation with $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$, we get $\tan\theta = \sqrt{3}$ and $u = 2\ \text{m/s}$.

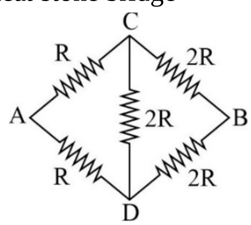
6.(A) $\frac{1}{\lambda} = R\left[1 - \frac{1}{n^2}\right]$

or $\frac{1}{\lambda R} = 1 - \frac{1}{n^2} \Rightarrow \frac{1}{n^2} = 1 - \frac{1}{\lambda R}$

or $\frac{1}{n^2} = \frac{\lambda R - 1}{\lambda R} \Rightarrow n^2 = \frac{\lambda R}{\lambda R - 1}$

or $n = \left(\frac{\lambda R}{\lambda R - 1}\right)^{1/2}$.

7.(C) It is wheat stone bridge



8.(A) Equating pressures, $\frac{F}{\pi R^2} = \frac{1200}{\pi[20R]^2} \Rightarrow F = 3kgf$

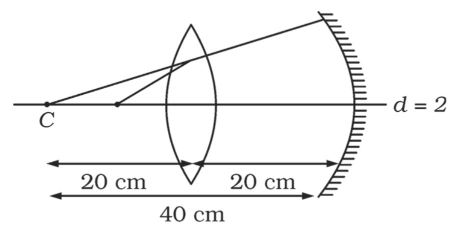
9.(C) The silvered lens can be replaced by a mirror of focal length given as

$\frac{1}{F_M} = \frac{1}{f_m} - \frac{2}{f_1}, \frac{1}{F_M} = 0 - \frac{2}{40}\ F_M = -20$

For lens $v = \frac{uf}{u + f'}\ v = \frac{-10 \times 20}{-10 + 20} = -20$

So, this position has to be centre of curvature of mirror in order for the ray to retrace its path so,

$d = 40 - 20 = 20cm$



10.(B) $T_1 = 27 + 273 = 300K$

$T_2 = 127 + 273 = 400K$

$\frac{E_2}{E_1} = \frac{T_2}{T_1} \Rightarrow E_2 = \frac{4}{3} \times 6.2 \times 10^{-21} J = 8.27 \times 10^{-21} J$

11.(D) $\frac{a_m}{g} = \frac{GM_e / R_m^2}{GM_e / R_e^2}\ or\ \frac{a_m}{g} = \frac{R_e^2}{R_m^2}\ or\ a_m = \left[\frac{R_e}{R_m}\right]^2 g$

12.(B) $U = \frac{\frac{3}{2}RT + \frac{6}{2}RT}{2} = \frac{9}{4}RT; C_v = \frac{dU}{dT} = \frac{9}{4}R$

$C_p = C_v + R = \frac{13}{4}R \Rightarrow \frac{C_p}{C_v} = \frac{13}{9} = 1.44$

13.(C) When a body is at highest position motion under gravity, then at that time velocity is zero and acceleration is g.

14.(C) $\lambda_{\min} = \dfrac{hc}{eV}$

15.(B) $\beta = \dfrac{3}{10} = 0.3cm$

$\beta = \dfrac{D\lambda}{d}$

$\therefore\ 0.3 = \dfrac{300 \times 5100 \times 10^{-8}}{d}$ or $d = \dfrac{300 \times 5100 \times 10^{-8}}{0.3} cm$

$= 51 \times 10^{-3} cm = 51 \times 10^{-2} mm = 0.51\, mm$

16.(A) $\vec{u} = 10\hat{i}\ m/s$

$\vec{a} = 2\hat{j}\ m/s^1$

$\vec{v} = \vec{u} + \vec{a}t$

$\vec{v} = 10\hat{i} + 2\hat{j} \times 5 = \left(10\hat{i} + 10\hat{j}\right) m/s$

$v = 10\sqrt{2} m/s = 14 m/s$

17.(C) $eV_s = hv - \phi_0$

$eV_s' = hv' - \phi_0$

$e\left(V_s' - V_s\right) = hv' - hv = \left(\dfrac{12375}{3600} - \dfrac{12375}{4000}\right) eV$

$\therefore\quad V_s' - V_s = 3.44 - 3.09 = 0.35V$

18.(B) The emf induced across the rod AB is

$e = Bv_{\perp} l$

Here, $v_{\perp} = v \sin 30°$ = component of velocity perpendicular to length

$\therefore\quad e = Bvl \sin 30°$

$= (2)(4)(1)\left(\dfrac{1}{2}\right) = 4V$

The free electrons of the rod shift towards right due to the force $q\left(\vec{v} \times \vec{B}\right)$. Thus, the left side of the rod is at higher potential.

or $\quad V_A - V_B = 4V$

19.(C) Force on block = $20 \times 2N = 40N$

Frictional force on block = $0.15 \times 20 \times 10N = 30N$

Net force = 10 N

Acceleration = $\dfrac{10N}{20kg} = \dfrac{1}{2} m\, s^{-2}$

Now, $4 = 0 \times t + \dfrac{1}{2} \times \dfrac{1}{2} t^2\ or\ t = 4s$

Let us calculate the distance travelled by the truck.

$x = 0 \times 4 + \dfrac{1}{2} \times 2 \times 4 \times 4 = 16m$

20.(B) Coefficient of x is angular wave number k of $\frac{2\pi}{\lambda}$.

Thus, $k = \frac{2\pi}{\lambda} = \pi \times 10^3$

$\therefore \quad \lambda = 2 \times 10^{-3} m$

NUMERICAL TYPE

1.(275) $P = E_v I_v \cos\phi$

$P = E_v \frac{E_v}{Z} \frac{R}{Z}$

or $\quad P = \frac{E_v^2 R}{Z^2} = \frac{110 \times 110 \times 11}{22 \times 22} W = 275 W.$

2.(9) $v = \frac{1}{2l}\sqrt{\frac{T}{m}}$ *or* $v \propto \frac{\sqrt{T}}{l}$ *or* $\sqrt{T} \propto vl$

$\therefore \sqrt{\frac{T_1}{T_2}} = \frac{100l}{75(2l)} = \frac{2}{3}$ *or* $\frac{T_1}{T_2} = \frac{4}{9}$

3.(3) $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ *or* $\frac{1}{\lambda} = R\left[\frac{1}{4} - \frac{1}{16}\right]$

$\frac{1}{\lambda} = R\left[\frac{4-1}{16}\right]$ *or* $\lambda = \frac{16}{3R}$

4.(6) Given, $q = 1200C, t = 20$ minute, A = 25 $mm^2 = 25 \times 10^{-6} m^2$

$n = 6.0 \times 10^{22}$ electrons/ $cm^3 = 6.0 \times 10^{22} \times 10^6 / m^3$

Drift velocity $v_d = ?$

We know that, $I = \frac{q}{t} = \frac{1200}{20 \times 60} = 1A$

Drift velocity, $v_d = \frac{I}{neA}$

$v_d = \frac{1}{6 \times 10^{22} \times 10^6 \times 25 \times 10^{-6} \times 1.6 \times 10^{-19}}$

$v_d = \frac{4.16 \times 10^{-3}}{10^3}$

$v_d = 4.2 \times 10^{-6} m/s$

5.(2880) Energy per unit volume $= \frac{1}{2} \times$ stress × strain = $\frac{1}{2}(Y\alpha t)(\alpha t) = \frac{1}{2} Y \alpha^2 t^2$

$= \frac{10^{11} \times 144 \times 10^{-12} \times 400}{2} J\ m^{-3}$

$= 288 \times 10 J\ m^{-3} = 2880\ J\ m^{-3}$

6.(5) $a = \dfrac{g\sin\theta}{1+\dfrac{K^2}{R^2}} = \dfrac{g\sin 30^\circ}{1+\dfrac{2}{5}}$ or $a = \dfrac{5}{7}g \times \dfrac{1}{2} = \dfrac{5g}{14}$

7.(4) Applying conservation of momentum, $1\times 4 = 2\times v$ *or* $v = 2ms^{-1}$

Loss of kinetic energy $= \dfrac{1}{2}\times 1\times 4\times 4 - \dfrac{1}{2}\times 2\times 2\times 2 = (8-4)\,J = 4J$

8.(9) Potential difference between A and B = 6 volts. The condensers $2\,\mu F$ *and* $5\mu F$ are in parallel. Their effective capacitance, C = 2 + 5 = 7 μF.

The capacitance between A and B is given by $C' = \dfrac{C\times 3}{C+3} = \dfrac{7\times 3}{7+3} = \dfrac{21}{10}\mu F$

Total charge $Q = CV = \dfrac{21}{10}\times 6 = \dfrac{63}{5}\mu C$

Total potential difference across $3\mu F$ is

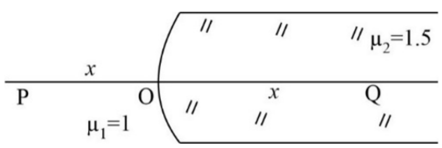
$V_1 = \dfrac{Q}{3} = \dfrac{63}{5}\times\dfrac{1}{3} = \dfrac{21}{5}$ volts

$V_2 = 6 - \dfrac{21}{5} = \dfrac{9}{5}V$

So, the charge on $5\,\mu F$ condenser is

$Q = 5\times V_2 = 5\times\dfrac{9}{5} = 9\mu C$

9.(5) $\dfrac{\mu_1}{-u} + \dfrac{\mu_2}{v} = \dfrac{\mu_2 - \mu_1}{R}$



$\dfrac{1}{-(-x)} + \dfrac{1.5}{x} = \dfrac{1.5-1}{R}$

$\dfrac{1}{x} + \dfrac{3}{2x} = \dfrac{1}{2R}$

$\dfrac{5}{2x} = \dfrac{1}{2R}$ *or* $x = 5R$

10.(42) Find at a point x from the centre of a current carrying loop on the axis is

$B = \dfrac{\mu_0}{4\pi}.\dfrac{2M}{x^3} = \dfrac{10^{-7}\times 2\times 2.1\times 10^{-25}}{\left(10^{-10}\right)^3}$

$= 4.2\times 10^{-32}\times 10^{30} = 4.2\times 10^{-2}W$

Chemistry

SINGLE CHOICE

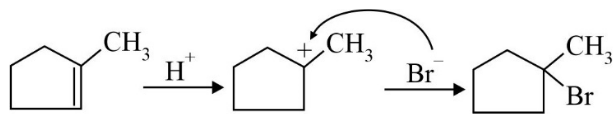
1.(A) The reaction that takes place is $NaCl + AgNO_3 \longrightarrow AgCl\downarrow + NaNO_3$

$\because$ 143.5g of AgCl is produced from 58.5g NaCl

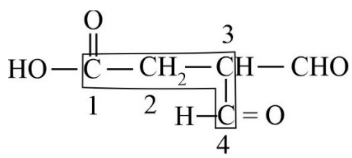
$\therefore$ 14g of AgCl will be produced from $\frac{58.5\times14}{143.5} = 5.70g\ NaCl$

5.70g is the amount of NaCl in common salt; % purity = $\frac{5.70}{6}\times100 = 95\%$

2.(D)



3.(B)



3 – formyl – 4 – oxo butanoic acid

4.(B) $w = -nRT\ln\frac{V_2}{V_1}$

$w = -2\times8.314\times10^{-3}\times300\ln\frac{40}{4},$

$w = -11.488kJ$

5.(C) Phenolphthalein is a weak organic acid and represented by Ph-H

$PhH \rightleftharpoons Ph^{(-)} + H^{(+)}$

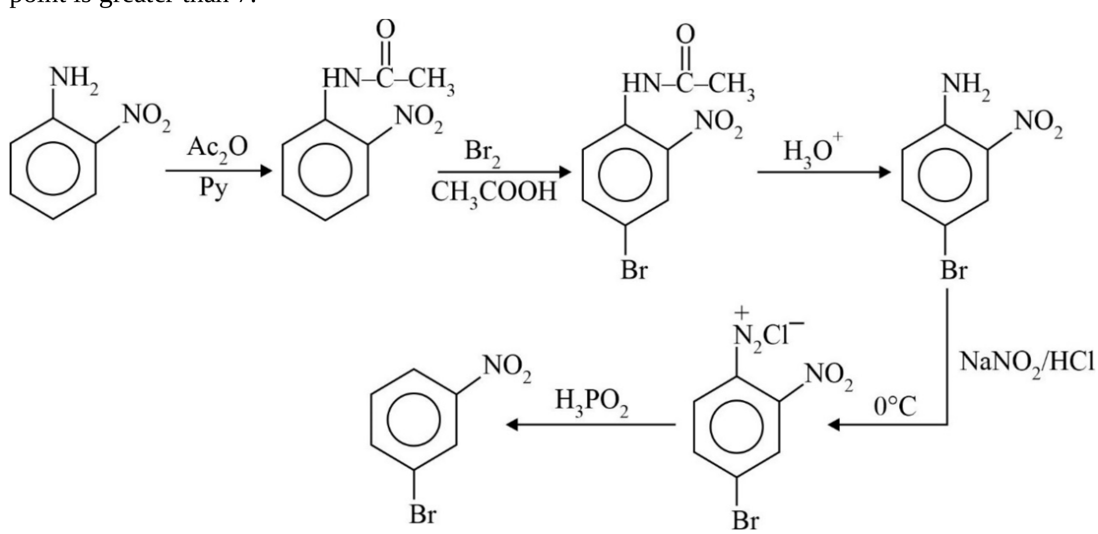
Colorless pink colour

Statement I- False

Statement II- False

6.(A) In redox titration, $KMnO_4$ act as self indicator and because of hydrolysis of anion, pH at equivalence point is greater than 7.

7.(C)



8.(B) More activation energy, slow reaction rate.

9.(B) According to Lewis, structure with complete octet of most of the 2^{nd} period element is the most stable.

10.(B) More percentage s-character means more electronegativity, more bond angle, less size and less energy.

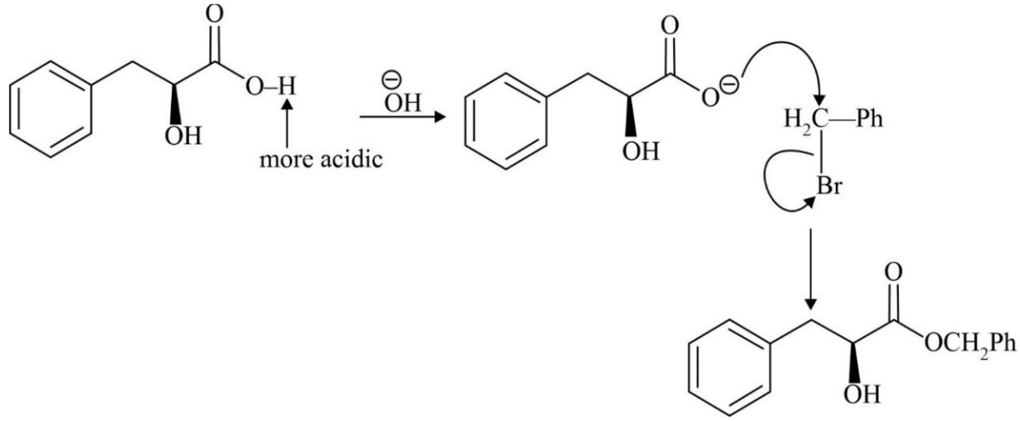
11.(D) $[NiCl_4]^{2-} = sp^3(4s + 4p_x + 4p_y + 4p_z)$

$[Ni(NH_3)_6]^{2+} = sp^3d^2(4s + 4p_x + 4p_y + 4p_z + 4d_{x^2-y^2} + 4d_{z^2})$

$[Ni(CN)_4]^{2-} = dsp^2(3d_{x^2-y^2} + 4s + 4p_x + 4p_y)$

$[Fe(CN)_6]^{3-} = d^2sp^3(3d_{x^2-y^2} + 3d_{z^2} + 4s + 4p_x + 4p_y + 4p_z)$

12.(A)



13.(C) Correct reducing character is $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$.

14.(D) IE $\rightarrow$ Mg $>$ Al

$1s^2 2s^2 2p^6 3s^2$ $\quad$ $1s^2 2s^2 2p^6 3s^2 3p^1$

Due to stable configuration of Mg.

Size : $Mg^{2+} > Al^{3+}$

More positive charge means more Z_{eff} so size decreases.

15.(A)

16.(D) Magnetic moment $= \sqrt{n(n+2)}$ BM

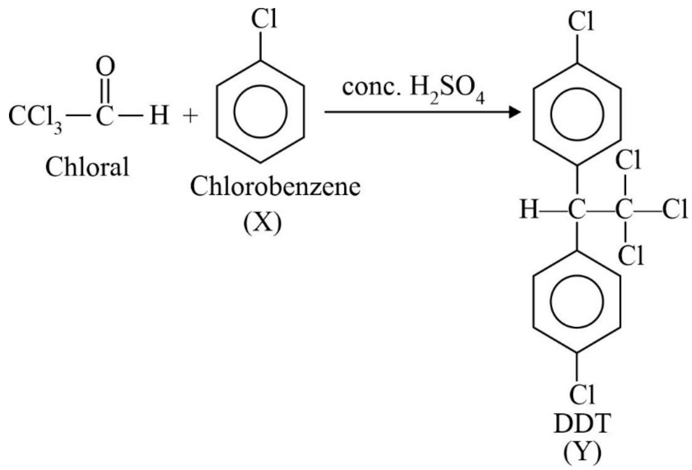
n : Number of unpaired e^-

As atomic number increases in d-block element number of unpaired e^- first increases upto 6 then decreases.

17.(C) Carboxylic acids are more acidic than alcohol. CCl_3 at $\alpha-$ position has greater E.W. effect than NO_2 at $\beta-$ position. Carbonyl F.G at $\beta-$ position has lesser E.W. effect than NO_2 group.

18.(A) More the electron withdrawing groups at ortho and para position more is the reactivity.

19.(B) For f-block elements common O.S. is +3.

20.(A) 

NUMERICAL TYPE

1.(6) Let solubility of Ag_2CO_3 in presence of Na_2CO_3 is S

$$Ag_2CO_3(s) \rightleftharpoons \underset{2S}{2Ag^+}(aq.) + \underset{(S+0.1)}{CO_3^{2-}}(aq.)$$

$$K_{sp} = \left[Ag^+\right]^2\left[CO_3^{2-}\right]$$

$$\Rightarrow\ 4\times10^{-13} = (2S)^2 \times 0.1$$

$$\Rightarrow\ S = 10^{-6}\ \Rightarrow\ x = 6$$

2.(0) For zero order reaction, $t_{1/2} \propto [\text{initial conc.}]^1$

3.(3) Angular node = ℓ

$7s \Rightarrow A.N = 0$

$7p \Rightarrow A.N = 1$

$6s \Rightarrow A.N = 0$

$8p \Rightarrow A.N = 1$

$8d \Rightarrow A.N = 2$

4.(1) $Q_v = 1.25\times4 = 5$

$$Q_v = 5 = \frac{x}{80}\times400\ \Rightarrow\ x = 1$$

5.(3) SF_3Cl, XeO_2F_2, SF_4

6.(1) $$\underset{S-x}{AB_2}(aq) \rightleftharpoons \underset{x}{A^{2+}}(aq) + \underset{2x}{2B^-}(aq)$$

$$K_{sp} = 2.56\times10^{-7} = 4x^3;\ x = 4\times10^{-3}M$$

$$\Delta T_b = K_b(S+2x)$$

$$6.5\times10^{-3} = 0.5\left(S + 2\times4\times10^{-3}\right);\ S = 5\times10^{-3}$$

Concentration of $AB_2(aq) = S - x = 5\times10^{-3} - 4\times10^{-3} = 1\times10^{-3}M$

7.(9) Sum of oxidation state = 5 + 0 + 4 = 9

8.(2) $\left[Pt(SCN)_2(OX)_2\right]^{x-}$

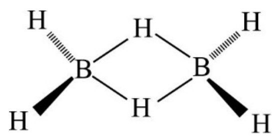
Number of unpaired electron = 0

So, Pt must be in +4 oxidation state $\left[Pt(SCN)_2(OX)_2\right]^{x-}$

$\Rightarrow 4+(-6)=-x$

$\Rightarrow x = 2$

9.(2) At cathode Reduction takes place so it may gain weight during reaction when metal is deposited after reduction. (A and B statements are correct)

10.(6) 

Mathematics

SINGLE CHOICE

1.(C) $(a-2)+ib=-\dfrac{(1+2\cos\theta)+2i\sin\theta}{(2+\cos\theta)+i\sin\theta}$

$\Rightarrow\ (a-2)^2+b^2=\dfrac{(1+2\cos\theta)^2+4\sin^2\theta}{(2+\cos\theta)^2+\sin^2\theta}=\dfrac{5+4\cos\theta}{5+4\cos\theta}=1$

$\lambda=1$

$\therefore\ -1\le 1-2\sin\alpha\le 3$

Range $(-\infty,-1]\cup\left[\dfrac{1}{3},\infty\right)$

2.(B) Coordinates of any point Q on the given line are $(2r+1,-3r-1,8r-10)$.

So the direction ratios of PQ are $2r,-3r-1,8r-10$.

Now PQ is perpendicular to the given line.

if $2(2r)-3(-3r-1)+8(8r-10)=0$

$\Rightarrow\quad 77r-77=0\Rightarrow r=1$

and the coordinates of Q, the foot of the perpendicular from P on the line are $(3,-4,-2)$.

Let $R(a,b,c)$ be the reflection of P in the given lines then Q is the mid-point of PR

$\Rightarrow\quad \dfrac{a+1}{2}=3,\dfrac{b}{2}=-4,\dfrac{c}{2}=-2$

$\Rightarrow\quad a=5,\ b=-8,\ c=-4$

and the coordinate of the required point are (5, –8, –4).

3.(D) As |A| = 0, we get $ad-bc=0$

Also, $A^2-(a+d)A=A[A-(a+d)I]$

$=\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}-d & b\\ c & -a\end{pmatrix}=\begin{pmatrix}-ad+bc & ab-ba\\ -cd+cd & bc-ad\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$

Thus, $-kI=0\Rightarrow k=0$

4.(C) $\vec{a}\times(2i+3j+4k)=(2i+3j+4k)\times\vec{b}=-\vec{b}\times(2i+3j+4k)$

$\Rightarrow\ (\vec{a}+\vec{b})\times(2i+3j+4k)=0$

$\Rightarrow\ (\vec{a}+\vec{b})$ is parallel to $(2i+3j+4k)=0$

$\Rightarrow(\vec{a}+\vec{b})=\alpha(2i+3j+4k)$ for some $\alpha\in R$

$\Rightarrow\left|\vec{a}+\vec{b}\right|=|\alpha|\sqrt{4+9+16}$

$\Rightarrow\ \alpha=\pm1$

Also, $(\vec{a}+\vec{b})\cdot(-7\hat{i}+2\hat{j}+3\hat{k})$

$= \alpha\left(2\hat{i}+3\hat{k}+4\hat{k}\right)\cdot\left(-7\hat{i}+2\hat{j}+3\hat{k}\right)$

$= \alpha\left(-14+6+12\right) = 4\alpha$

Taking $\alpha = 1,$ we get one of the possible values as 4.

5.(A) As we are interested in coefficient of $t^{50},$ we shall ignore all the term with exponent more than 50. Thus, we can write given expression as

$\left(1+{}^{25}C_1 t^2+...+{}^{25}C_{25} t^{50}\right)\times\left(1+t^{25}+t^{40}+t^{45}+t^{47}\right)$

As all the terms in the first expression have even exponent we can ignore $t^{25}, t^{45} and\ t^{47}$ too. Thus, coefficient of t^{50} in (1) $= {}^{25}C_{25} + {}^{25}C_5 = 1 + {}^{25}C_5$

6.(A) Equation of the circle C is $(x-2)^2+(y-1)^2 = r^2$

$\Rightarrow x^2+y^2-4x-2y+5-r^2 = 0$

Equation of the common chord is $\left(x^2+y^2-4x-2y+5-r^2\right)-\left(x^2+y^2-2x-6y+6\right)=0$

$\Rightarrow 2x-4y+r^2+1=0$

If it is a diameter of the second circle, it passes through the centre (1, 3) of the circle.

So $2-4\times3+r^2+1=0 \Rightarrow r^2=9 \Rightarrow r=3$

7.(A) Let *d* be the common difference. Then $\log_y x = 1+d \Rightarrow x = y^{1+d}$

$\log_z y = 1+2d \Rightarrow y = z^{1+2d}$

and $-15\log_x z = 1+3d \Rightarrow z = x^{-(1+3d)/15}$

$x = y^{1+d} = z^{(1+2d)(1+d)}$

$= x^{-(1+d)(1+2d)(1+3d)/15}$

$\Rightarrow (1+d)(1+2d)(1+3d) = -15$

$\Rightarrow 6d^3+11d^2+6d+16=0$

$\Rightarrow (d+2)\left(6d^2-d+8\right)=0 \Rightarrow d=-2$

[Note that $6d^2-d+8=0$ has complex roots]

$\therefore \quad x = y^{1+d} = y^{-1}, y = z^{1-4} = z^{-3}$ and $x = \left(z^{-3}\right)^{-1} = z^3.$

8.(D) $f(0) = \lim_{x\to0} f(x)$

$= \lim_{x\to0}\frac{(1+x)\log(1+x)-x}{x^2}\ \left(\frac{0}{0}\ form\right)$

$= \lim_{x\to0}\frac{\log(1+x)+1-1}{2x}$

$= \frac{1}{2} \lim_{x \to 0} \frac{\log(1+x)}{x} = \frac{1}{2}$

$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$ (As given expression is an AGP)

$= \frac{1}{1/2} + \frac{1/2}{1/4} = 4$

9.(B) (M, M), (A, A), (T, T), E, H, I, C, S

We can arrange 4 distinct letters in ${}^8P_4$ ways, 2 identical and 2 distinct in $\left({}^3C_1\right)\left({}^7C_2\right)\frac{4!}{2!}$ ways and 2 pairs of identical letters in $\left({}^3C_2\right)\frac{4!}{2!2!}$ ways

10.(D) If A denotes the coefficient matrix, then $|A| = \lambda^2(\lambda+3)$

For $\lambda = 0$ *and* $\lambda = -3$, the system is inconsistent.

$\tan\theta = 0$ $\qquad$ $\tan\theta = -1$

$\Rightarrow \theta = 0, \pi, 2\pi$ $\qquad$ $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

$S = \frac{11\pi}{2}$

11.(A) We have $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \cos^2\alpha + \cos^2\left(\frac{\pi}{2} - \alpha\right) + \cos^2\gamma = 1$

$\Rightarrow \cos^2\alpha + \sin^2\alpha + \cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = 0 \Rightarrow \cos\gamma = 0$

So $(\cos\alpha + \cos\beta + \cos\gamma)^2 = (\cos\alpha + \sin\alpha)^2 = 1 + 2\sin\alpha\cos\alpha$

$= 1 + \sin 2\alpha$

12.(C) Suppose the bisector of angle A meets BC at D. Then AD divides BC in the ratio AB : AC. So, P.V. of D is given by

$$\frac{\left|\overrightarrow{AB}\right|\left(2\hat{i} + 5\hat{j} + 7\hat{k}\right) + \left|\overrightarrow{AC}\right|\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)}{\left|\overrightarrow{AB}\right| + \left|\overrightarrow{AC}\right|}$$

But $\overrightarrow{AB} = -2\hat{i} - 4\hat{j} - 4\hat{k}$

and $\overrightarrow{AC} = -2\hat{i} - 2\hat{j} - \hat{k}$

$\Rightarrow \left|\overrightarrow{AB}\right| = 6$ *and* $\left|\overrightarrow{AC}\right| = 3$

Therefore, P.V. of D is given by

$$\frac{6\left(2\hat{i} + 5\hat{j} + 7\hat{k}\right) + 3\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)}{6+3} = \frac{1}{3}\left(6\hat{i} + 13\hat{j} + 18\hat{k}\right)$$

13.(D) $\left(\frac{1}{x^2}-x^3\right)^n$

General term $T_{r+1}=\frac{n!}{r!(n-r)!}(-1)^r x^{5r-2n}$

If $5r-2n=5$, then $5r=2n+5$ or $r=\frac{2n}{5}+1$

If $5r-2n=10$, then $5r=2n+10$ or $r=\frac{2n}{5}+2$

x^5 *and* x^{10} terms occurs if $n=5k$

Given that sum of x^5 *and* x^{10} is zero.

$\Rightarrow \quad \frac{5k!}{(2k+1)!(3k-1)!}-\frac{5k!}{(2k+2)!(3k-2)!}=0$

or $\quad \frac{1}{3k-1}-\frac{1}{2k+2}=0$

or $\quad k=3 \Rightarrow n=15$

$\left(x^2+\frac{1}{x^2}+2\right)^{15}$

$\left(x+\frac{1}{x}\right)^{30}$

Term independent of x is 16th term, $T_{16}={}^{30}C_{15}=\frac{30!}{(15!)^2}$

14.(A) Reflexive but not symmetric

$\because \quad x^2-3xy+2y^2=0$

$\Rightarrow \quad x^2-xy-2xy+2y^2=0$

$\Rightarrow \quad x(x-y)-2y(x-y)=0$

$\Rightarrow \quad (x-2y)(x-y)=0$

$\Rightarrow \quad x=y \;\; or \;\; x=2y$

Now, $\because$ in R all ordered pairs (x, x) are present

$\therefore$ It is reflexive

Now, $(4,2)\in R$ as 4 = 2 (2)

but (2, 4) $\notin$ R as $2\neq 2(4)$

$\therefore$ It is not symmetric

Also (4, 2) and (2, 1) $\in R$ but $(4,1)\notin R$

$\therefore$ It is not transitive

15.(C) $\frac{{}^{20}C_2+{}^{19}C_2}{{}^{39}C_3}=\frac{19}{481}$

Number of APs = Number of ways of selecting two numbers whose sum is even.

16.(B) $\int\left[a^x \ln x + \underbrace{a^x \ln a}_{II}.\underbrace{x(\ln x - 1)}_{I}\right]dx$

$$= \int a^x.\ln x\,dx + \left[x(\ln x - 1)a^x - \int\left[x.\frac{1}{x} + (\ln x - 1)\right]a^x\right]dx$$

$$= \int a^x.\ln x\,dx + \left[x(\ln x / e)\right]a^x - \int(\ln x)a^x dx$$

17.(A) $\left(2\sin^2 91° - 1\right)\left(2\sin^2 92° - 1\right)....\left(2\sin^2 180° - 1\right)$, In this product there exists a factor $\left(2\sin^2 135° - 1\right)$, which is equal to zero. Thus, the product of all terms is zero.

18.(B) Let $P \equiv \left(at_1^2, 2at_1\right)$ *and* $Q \equiv \left(at_2^2, 2at_2\right)$.

$\Rightarrow \quad t_1 t_2 = -1$

Equation of AP is $y = \frac{2}{t_1}x_1$.

$\Rightarrow R \equiv \left(-a, -\frac{2a}{t_1}\right)$

Similarly, $T \equiv \left(-a, \frac{-2a}{t_2}\right)$

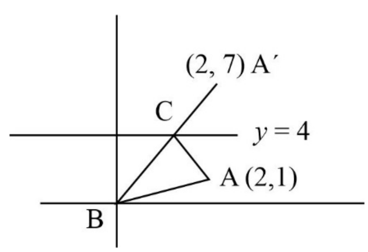
Slope of RS = $\dfrac{\frac{(-2a)}{t_1}}{-2a} = \frac{1}{t_1}$

Slope of TS = $\dfrac{\frac{(-2a)}{t_2}}{-2a} = \frac{1}{t_2}$

Now, $\left(\frac{1}{t_1}\right) \times \left(\frac{1}{t_2}\right) = \frac{1}{t_1 t_2} = -1$

$\Rightarrow \quad \angle RST = 90°$

19.(D) Clearly BC + AC should be min (for perimeter P to be minimum) Take image of A in y = 4, say it is $A'(2, 7)$



$\therefore$ now AC = $A'C$

If $C = (t, 4)$ then $\frac{4}{t} = \frac{7}{2} \Rightarrow t = \frac{8}{7} = \beta$ (Slope of BC = CA′)

for P to be max C should be at (4, 4)

i.e., $t = 4$

$\Rightarrow \alpha = 4$

$\therefore \quad = 6\alpha + 21\beta = 24 + 24 = 48$

20.(D) $\bar{x}_{new} = \frac{4}{3}\bar{x}$

From given information $\bar{x}_{new} = \frac{n\bar{x} + 2k\bar{x}}{n+k} = \frac{4}{3}\bar{x}$

$\Rightarrow 3n + 6k = 4n + 4k$

$\Rightarrow 2k = n$

$\Rightarrow \frac{k}{n} = 0.5$

NUMERICAL TYPE

1.(997) Given, $f(x) = \frac{9^x}{9^x + 3}$ …(i)

$$\Rightarrow \quad f(1-x) = \frac{9^{1-x}}{9^{1-x} + 3}$$

$$\Rightarrow \quad f(1-x) = \frac{\frac{9}{9^x}}{\frac{9}{9^x} + 3} = \frac{9}{9 + 3 \cdot 9^x}$$

$$f(1-x) = \frac{9}{3(3 + 9^x)} \quad \text{…(ii)}$$

On adding Eqs. (i) and (ii), we get

$$f(x) + f(1-x) = \frac{9^x}{9^x + 3} + \frac{9}{3(3 + 9^x)}$$

$$= \frac{3 \cdot 9^x + 9}{3(9^x + 3)} = \frac{3(9^x + 3)}{3(9^x + 3)}$$

$\therefore \quad f(x) + f(1-x) = 1$ …(iii)

Now, putting $x = \frac{1}{1996}, \frac{2}{1996}, \frac{3}{1996}, \ldots\ldots, \frac{998}{1996}$ in Eq. (iii),

we get $f\left(\frac{1}{1996}\right) + f\left(\frac{1995}{1996}\right) = 1$

$$\Rightarrow \quad f\left(\frac{2}{1996}\right) + f\left(\frac{1994}{1996}\right) = 1$$

$$\Rightarrow \quad f\left(\frac{3}{1996}\right) + f\left(\frac{1993}{1996}\right) = 1$$

……………………………………

……………………………………

$$\Rightarrow \quad f\left(\frac{997}{1996}\right) + f\left(\frac{999}{1996}\right) = 1$$

$\Rightarrow \quad f\left(\frac{998}{1996}\right)+f\left(\frac{998}{1996}\right)=1$

$\Rightarrow \quad f\left(\frac{998}{1996}\right)=\frac{1}{2}$

On adding all the above expressions, we get

$$k=f\left(\frac{1}{1996}\right)+f\left(\frac{2}{1996}\right)+...+f\left(\frac{1995}{1996}\right)$$

$$=\underbrace{(1+1+1........+1)}_{997\text{ times}}+\frac{1}{2}$$

$$k=997+\frac{1}{2}$$

$$\therefore \quad \left|k-\frac{1}{2}\right|=\left|997+\frac{1}{2}-\frac{1}{2}\right|=997$$

2.(6) $2\left(x_2x_3+x_3x_1+x_1x_2\right)=\left(x_1+x_2+x_3\right)^2-\left(x_1^2+x_2^2+x_3^2\right)=-2$

$x_1^3+x_2^3+x_3^3-3x_1x_2x_3$

$=\left(x_1+x_2+x_3\right)\left(x_1^2+x_2^2+x_3^2-x_2x_3-x_3x_1-x_1x_2\right)$

$=(2)(6+1)=14$

$\Rightarrow x_1x_2x_3=-2.$

Equation whose roots are x_1, x_2, x_3 is $x^3-2x^2-x+2=0 \Rightarrow x=2,1,-1.$

Thus, $\left(x_2-x_3\right)\left(x_3-x_1\right)\left(x_1-x_2\right)=6.$

3.(7) Midpoint of the line segment is $\left(\frac{1-3}{2},\frac{2+6}{2},\frac{6+2}{2}\right)\equiv(-1,4,4)$

Parallel vector to the required line $=(-1+6)\hat{i}+(4-2)\hat{j}+(4-4)\hat{k}=5\hat{i}+2\hat{j}+0\hat{k}$

Hence, equation of the line is $\frac{x+6}{5}=\frac{y-2}{2}=\frac{z-4}{0}$

$\Rightarrow l=5, m=2, n=0$

4.(4) $(-\infty,-3]\cup(-2,-1)\cup(2,\infty)$

$\log_{(|x|-1)}\left(x^2+4x+4\right)\geq 0$

Case 1: $0<|x|-1<1$

i.e., $1<|x|<2,$ then $0<x^2+4x+4\leq 1 \Rightarrow x^2+4x+3\leq 0\ \&\ (x+2)^2\neq 0$

$\Rightarrow -3\leq x\leq -1\ \&\ x\neq -2$

So, $x\in(-2,-1)$

Case 2 : $|x|-1>1$

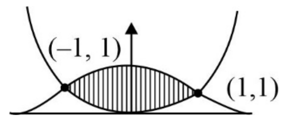
i.e., $|x| > 2$

$x^2 + 4x + 4 \geq 1 \Rightarrow (x+1)(x+3) \geq 0 \Rightarrow x \in (-\infty, -3] \cup [-1, \infty]$

$\Rightarrow x \in (-\infty, -3] \cup (2, \infty)$

Hence, domain is $(-\infty, -3] \cup (-2, -1) \cup (2, \infty)$

5.(3) Drawing graphs of $y = x^2$ *and* $y = \dfrac{2}{x^2+1}$



For points of intersection, $x^2 = \dfrac{2}{x^2+1} \Rightarrow x^4 + x^2 - 2 = 0$

$x^2 = 1 \Rightarrow x = \pm 1$

Required area $= 2\int_0^1 \left(\dfrac{2}{1+x^2} - x^2\right)dx = \left(\pi - \dfrac{2}{3}\right) sq.$ units

$K_1 = 1, K_2 = 2 \Rightarrow K_1 + K_2 = 3$

6.(9) $a-d, a, a+d, a-d+30$

If last three terms are in G.P. $(a+d)^2 = a(a-d+30)$

$\Rightarrow \quad a = \dfrac{d^2}{30-3d}$

$\Rightarrow$ For a to be integer $d = 9$

7.(4) The given equation can be written as $\dfrac{dy}{dx} - y\dfrac{\phi'(x)}{\phi(x)} = \dfrac{-y^2}{\phi(x)}$

$\Rightarrow \quad -\dfrac{1}{y^2}\dfrac{dy}{dx} + \dfrac{1}{y}\dfrac{\phi'(x)}{\phi(x)} = \dfrac{1}{\phi(x)}$

Put $\dfrac{1}{y} = u$ so that $-\dfrac{1}{y^2}\dfrac{dy}{dx} = \dfrac{du}{dx}$. Now (1) becomes $\dfrac{du}{dx} + u\dfrac{\phi'(x)}{\phi(x)} = \dfrac{1}{\phi(x)}$

This is a linear equation in *u* so its integrating factor is $e^{\int \frac{\phi'(x)}{\phi(x)}dx} = e^{\log \phi(x)} = \phi(x)$

Multiplying both sides by the integrating factor, we have

$\dfrac{d}{dx}[u\phi(x)] = 1 \Rightarrow u\phi(x) = x +$ constant $\Rightarrow y = \dfrac{\phi(x)}{x+C}$

$\Rightarrow \quad y(1) = \dfrac{1}{1+C} \Rightarrow C = 0$. Thus $y = \dfrac{\phi(x)}{x}$

Hence $y(4) = \dfrac{1296}{4} = 324$

8.(100) Let $f(x)=\sin x+\cos x$. By considering the minimum and maximum value of

$f(x)\ on \left[0,\frac{\pi}{2}\right],\left[\frac{\pi}{2},\frac{3\pi}{4}\right],....\left[\frac{7\pi}{4},2\pi\right]$

$$[f(x)]=\begin{cases} 1 & 0\le x\le \frac{\pi}{2} \\ 0 & \frac{\pi}{2}<x\le \frac{3\pi}{4} \\ -1 & \frac{3\pi}{4}<x\le \pi \\ -2 & \pi<x<\frac{3\pi}{2} \\ -1 & \frac{3\pi}{2}\le x<\frac{7\pi}{4} \\ 0 & \frac{7\pi}{4}\le x<2\pi \end{cases}$$

So, $\int_0^{2\pi}[\sin x+\cos x]\,dx=\int_0^{\pi/2}dx+\int_{\pi/2}^{3\pi/4}0\,dx$

$+\int_{3\pi/4}^{\pi}(-1)\,dx+\int_{\pi}^{3\pi/2}(-2)\,dx+\int_{3\pi/2}^{7\pi/4}(-1)\,dx+\int_{7\pi/4}^{2\pi}0\,dx$

$=\frac{\pi}{2}+\frac{3\pi}{4}-\pi+2\pi-3\pi+\frac{3\pi}{2}-\frac{7\pi}{4}=-\pi$

Since $\sin x+\cos x$ is a periodic function with period 2π, $\int_0^{200\pi}[\sin x+\cos x]\,dx=-100\pi$

9.(4) For maximum number of common chords, we must have maximum number of common points. For y-coordinate of the point of intersections, we solve $4y+(y-7)^2=r^2$

or $\quad y^2-10y+49-r^2=0$

This equation will have two distinct real roots if $100-4\left(49-r^2\right)>0$

$\Rightarrow \quad r^2>24$

$\Rightarrow \quad r>\sqrt{24}$

$\Rightarrow$ least value of $[r]$ is 4.

10.(2) Since $g(x)=f^{-1}(x)$ so $f(g(x))=(x)$ for all *x*. Differentiating $f'(g(x))g'(x)=1$.

Let $g(1)=k$, then $f(g(1))=f(k)=1$. Since $f(0)=1\ and\ f$ is continuous and increasing function so one-one thus $k=0$. Putting $x=1$ in (i), we get $f'(0)g'(1)=1$. But $f'(x)=3x^2+(1/2)e^{x/2}\Rightarrow f'(0)=\frac{1}{2}$

$\therefore \quad \left(\frac{1}{2}\right)g'(1)=1\Rightarrow g'(1)=2.$